Prohibiting Dogmatism

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Abstract. Despite the intuitive appeal of a prohibition against extremal credences towards the unobserved, existing arguments in favour of such a prohibition have been systematically refuted. In this paper, I propose a new argument, which captures and elaborates on the crux of these existing arguments, and which avoids objections to them.

1.

You have received a book as a gift, which could be a novel or a poetry collection. Prior to unwrapping the book, what credence would it be rational for you to have in the two relevant propositions, *novel* and *poems*? Many have found a partial answer to this question plausible: it would not be rational for you to have credence 0 or 1 in either of these propositions—doing so would amount to a kind of dogmatism. My aim in this paper is to capture and vindicate this partial answer, which I shall call the *plausible thought*.

According to Bayesians, an agent's epistemic attitude can be represented by a function $p : \mathcal{A} \to [0,1]$, which assigns a credence to each proposition $A_i \in \mathcal{A}$ that the agent entertains. It is standardly assumed that the set $\mathcal{A}$ forms a *Boolean algebra* of some non-empty set Ω: $\mathcal{A}$ contains Ω and $\emptyset$, and is closed under negation and union.[1] I leave the interpretation of these objects for later on, but it will be relevant to note that the elements of $\mathcal{A}$ can be separated into two groups: the *trivial* propositions ($\emptyset$ and Ω), and the *non-trivial* ones ($A_1, A_2, ...$). It will also be relevant to note that credal values can be separated into two groups: *extremal* credences (0 and 1) and *non-extremal* ones (the others).

Two standardly accepted credal norms will be of interest to us.[2] Where $\mathcal{E} \subseteq \mathcal{A}$ is the set of all propositions which constitute the agent's evidence, said agent's credences ought to be such that:

> *Trivial omniscience.* $p(\Omega) = 1$ and $p(\emptyset) = 0$.
> *Evidential omniscience.* $p(A_{E_i}) = 1$ and $p(\neg A_{E_i}) = 0$, for all evidential propositions $A_{E_i} \in \mathcal{E}$.

[1] This assumption captures the thought that entertained propositions are governed by classical logic, and ensures that credences over them can be probabilistic.

[2] *Probabilism* is the conjunction of trivial omniscience and an additivity requirement; *conditionalisation* is the conjunction of evidential omniscience and a rigidity requirement.

We are now in a position to give a somewhat non-standard formulation of the principle which is usually taken to capture the plausible thought:

> ***Regularity.*** $p(A_i) \neq 1$ and $p(A_i) \neq 0$, for all $A_i \in \mathcal{A}$ such that $A_i \neq \Omega$ and $A_i \neq \emptyset$.

Regularity so-defined is the converse of trivial omniscience: where trivial ominiscience mandates extremal credences in trivial propositions, regularity prohibits non-extremal credences in non-trivial propositions.

Now, as Hájek (2012) points out, regularity stands in tension with evidential omniscience. Indeed, evidential omniscience mandates extremal credences in evidential propositions, but regularity prohibits this: evidential propositions are non-trivial. Thus, accepting regularity requires either (1) rejecting evidential omniscience, and thus, conditionalisation; or (2) asserting that there are no evidential propositions.[3] Alternatively, we could capture the plausible thought with a weaker norm:

> ***Humility.*** $p(A_i) \neq 1$ and $p(A_i) \neq 0$, for all $A_i \in \mathcal{A}$ such that $A_i \neq \Omega, \emptyset$, and $A_i \notin \mathcal{E}$.

Since options (1) and (2) constitute significant departures from Bayesian oxthodoxy, I shall assume for now that humility is the best way to capture the plausible thought; and I will return to the question later.

2.

The most widely discussed problem for humility is that, in Lewis' words, "there are too many alternative possible worlds to permit" it (1980, p. 267). This objection is made famously vivid by Bernstein and Wattenberg (1969) and popularised by Hájek (2003). They instruct their reader to consider an infinitely thin-tipped dart, thrown at a dart board; then ask: what credence ought one assign to the proposition that the dart will land on any given point? Given the requirement that credences be probabilistic, the agent cannot but flout humility: there is not enough credal mass to leave no point exposed. More precisely, the following three claims are mutually inconsistent: (a) that $\mathcal{A} \backslash \mathcal{E}$ may be uncountable, (b) that credences must be real-valued, and (c) humility. In response to this trilemma, Hájek (2003, 2012, ms) and Easwaran (2014) prominently advocate for a move away from the standard Bayesian framework in which agents' epistemic attitudes are represented by unconditional probabilities. But the majority of Bayesians instead reject (b)—though the legitimacy of this move is disputed by Easwaran (2014)—or (c) humility.

Here, I want to explore another type of response, related to but different from the rejecting of (a). As far as I know, no one has seriously argued against (a); that is, argued that agents may not consider uncountably-many propositions. Nor will I here. But in the example with which I started this paper, the agent considered a finite number of mutually exclusive propositions—***novels*** and ***poems***.

[3] Another option would be to restrict the domain of these norms to superbaby/prior credences. This option is extensionally equivalent to accepting humility (below), given the rejection of options (1) and (2). Furthermore, it presupposes the well-foundedness of superbaby idealisation—a substantive commitment in need of defence. For these reasons, I set it aside in this paper. Thanks to [redacted] for alerting me to it.

The question was whether it would be permissible to have extremal credences in these propositions. This highlights the possibility that, although humility might be false in general, it may nonetheless be true in those cases where the agent's algebra is countable. For this possibility to be attractive, it would have to be that the reasons we have against humility on uncountable algebras are not also reasons against it on countable ones. And indeed, this is what we have here: the reason we have to reject humility in uncountable cases is that it stands in tension with a basic Bayesian commitment to real-valued credences—but it does not in countable cases, where there is enough probabilistic mass to satisfy humility. This separation of countable and uncountable cases is sustained further by the fact that most Bayesians have differing intuitions across them: the assignment of an extremal credence to *poems* seems egregious, in a way that the assignment of such a credence to the proposition that the dart will land exactly at the centre of the board does not. Let us proceed then with humility, restricted to countable cases.

3.

Although humility—restricted to countable cases—may be intuitively appealing, it is surprisingly difficult to provide a convincing argument in its favour. In this section, I sample what I take to be the three most convincing attempts,[4] and I bring out the single consideration at the crux of their failures. The three arguments have a similar structure. They start off by drawing out a consequence of flouting humility. So, take an agent with credence 1 in *poems* and credence 0 in *novels*. Shimony (1955) remarks that this agent is liable to a loss, if *poems* turns out false; but does not stand to gain anything, if it turns out true.[5] Williamson (2007) remarks that it would be rational for this agent to bet their life on *poems*.[6] Finally, Lewis (1980) remarks that this agent knows that they cannot subsequently learn *novel*: given (standard) conditionalisation, a proposition can only be learnt if it is assigned a non-zero credence. The second step in these arguments is to insist that these consequences are undesirable. And the third step is to fault the failure of humility.

There is a common problem with these arguments, which Hájek identifies in relation to the first. He says: "an omniscient God who knows which world is actual, and so concentrates all credence on that world, is [as described by Shimony], and none the worse for that!" (2012, p. 420). This objection extends to Williamson's and Lewis's arguments, too: an omniscient God would be entirely justified in betting her life on the actual world; and she would be not be irrational for not being able to learn anything else. This analysis shows two things. The first is that an argument for humility cannot appeal to the consequences of doing so. And the second is that it must be sensitive to the difference between our book-receiving agent and God. I attempt to build such an argument in the rest of this paper.

[4] For a more comprehensive overview, see Hájek (2012).
[5] This argument structure is known as a *semi-Dutch Book*.
[6] This is a slight modification of Williamson's remark, which makes his argument stronger.

4.

So far, I have made minimal interpretive assumptions: I have assumed that credence functions represent epistemic attitudes, without saying what it means to say that these attitudes are rational, and I have discussed various features of rational credence functions in a purely formal manner. In the rest of this paper, I will fill in these gaps in a particular way, and show that humility ensues. This immediately raises the question of whether to accept my meta-epistemological claim. I should note upfront that will not attempt a foolproof defence of it in this short paper. I will however discuss what is at stake in whether to accept it, and what my arguments entail for those who do not.

To begin: consider the propositions *book or not*—that the object you have received is either a book or not; *book*—that the object you have received is a book; and *poems*—that the object you have received is a book of poems. Before unwrapping, you can settle the first two propositions: you can determine (by reasoning) that *book or not* is true, and you can determine (by testimony) that *book* is true. By contrast, you cannot settle the proposition *poems*: only after you have unwrapped the present will you be able to determine whether *poems* is true. These remarks suggest a particular account of epistemology, which I call *inquisitive.* On this account, the goal of epistemic activity is to determine what is the case, on the basis of one's means of inquiry. Sometimes this can be accomplished, and sometimes it cannot.

In the rest of this section, I provide an interpretation of the Bayesian formalism—and thus, of humility—that is in line with the inquisitive approach to epistemology. I start with the distinction between extremal and non-extremal credences. If the ultimate goal of epistemic activity is to settle propositions, the distinction between settled and provisional epistemic attitudes is highly salient. The Bayesian formalism offers an apt tool to capture this distinction: extremal and non-extremal credences. So, on this interpretation, extremal credences are settled attitudes, and non-extremal credences are provisional attitudes.

Furthermore, epistemic activity on the inquisitive approach involves the exercising of inquisitive means. These means can be divided into two types: the theoretic (reason) and the empiric (observation capaciously understood to include i.e. experience, testimony, etc.). This bipartition yields a corresponding bipartition among entertained propositions, consisting of those propositions that can be settled *a priori* on the one hand, and those that must be settled *a posteriori* on the other. Once again the Bayesian formalism is apt to model this distinction: the trivial propositions $\emptyset$ and Ω represent the propositions that can be settled *a priori*; the non-trivial ones $A_1, A_2, \ldots$ represent those that cannot—that must be settled *a posteriori.* Additionally, a further distinction must be made among propositions about the *a posteriori*, between the propositions which can be settled on the basis of the agent's observations to date, namely evidential propositions; and those which cannot, namely propositions about the unobserved. Again, this distinction can be captured by the Bayesian formalism: propositions $A_{E_1}, A_{E_2}, \ldots$ are evidential propositions; propositions $A_{U_1}, A_{U_2}, \ldots$ are about the unobserved.

This completes my inquisitive interpretation of Bayesian mathematics. Epistemic attitudes are divided into two types: settled and provisional. Propositions in which these attitudes are had are divided into three types: non-empirical propositions, propositions about the observed, and propositions about the (empirical) unobserved. With such an interpretation of basic components, we can interpret widespread putative norms of rationality; as concerns us here: humility.[7] Formally, humility is the claim that agents ought not have extremal credences in non-trivial non-evidential propositions. Interpreted as per the inquisitive approach, humility becomes the claim that agents ought not have settled attitudes towards the empirical unobserved.

This definition seems to me superior to the widespread definitions, of which there are two.[8] On the logical interpretation of the formalism, the trivial/non-trivial distinction is interpreted as the logically-necessary/logically-contingent distinction; and on the metaphysical interpretation, it is interpreted as the metaphysically-necessary/metaphysically-contingent distinction. Both of these interpretations yield versions of humility whose recommendations on propositions that are (metaphysically and therefore logically) contingent *a priori* differ from those of the inquisitive interpretation.[9] Indeed, humility on the logical and metaphysical interpretations prohibits credence 1 in the proposition that—to take Kripke's (1980) example—the standard metre stick measures one meter. The fact that the standard metre stick measures one meter is metaphysically and logically contingent—it could have not measured one meter—and the logical/metaphysical versions of humility disallow extremal credences in logically/metaphysically contingent propositions. But the proposition that the standard meter stick measures one meter is also determinable *a priori*—one needs not inquire into the world to know that it is true. As such, humility on the inquisitive approach allows agents to have credence 1 in it, as per our intuitions.[10]

5.

Why then accept humility?—Why prohibit settled attitudes in propositions about the (empirical) unobserved? This will depend on what it means to say that a particular epistemic attitude is rational. Now, the inquisitive approach highlights that epistemic agents are involved in epistemic *activity*; this suggests that to be rational on this approach is to perform this activity *well*. Since epistemic activity consists in wielding one's means of inquiry with the goal of determining what is the case, to be rational must be for one's epistemic attitudes to be connected in the right way to one's means of inquiry. What the right way is precisely is a complicated matter. But what is clear is that the settling of a proposition without the exercising of one's means of inquiry constitutes a breaking of the normative link between the means and the ends of inquiry, and as such, is in tension with the nature of epistemic rationality. This is the mistake made by the flouter of humility. A proposition about the empirical unobserved

[7] Mahtani (forthcoming) discusses how the interpretation of the trivial/non-trivial distinction impacts Dutch Book and accuracy arguments for probabilism.

[8] These definitions are given for regularity and adapted here for humility. Beyond them, other interpretations—epistemic and doxastic ones—are discussed for instance by Tang (2012) and Easwaran (2014). I will only note here that these assume the existence of epistemic attitudes beyond credences—an assumption which many orthodox Bayesians would reject.

[9] Conversely, probabilism will yield different recommendations on the logical and metaphysical interpretations than on the inquisitive interpretation, on propositions that are necessary *a posteriori*. See Chalmers (2011).

[10] This highlights that my version of humility is not trans-modal, unlike standard versions.

cannot, by definition, be settled by reason or one's observations to date. So, to have settled such a proposition is to have failed at epistemic activity; it is to have broken the link between means of inquiry and epistemic attitudes.

We are now in a position to return to the dispute between humility and regularity. Remember, regularity is stronger than humility: it imposes a supplementary constraint on credences, namely that evidential propositions ought not be assigned extremal credences. Interpreted inquisitively, this supplementary constraint prohibits settling propositions about the observed. Thus the debate between humility and regularity hinges on a question in the epistemology of perception, namely, that of when (if ever) we can determine how the empirical world is on the basis of our senses. This sheds new light on a remark I made above. I wrote that accepting regularity requires either (1) rejecting evidential omniscience, or (2) asserting that there are no evidential propositions. We can now understand why: accepting regularity requires claiming that we are never in a position to settle empirical facts on the basis of our senses.[11] Whether this holds goes far beyond the scope of this paper; what I hope to have shown is that humility is true.

6.

I have argued that on countable algebras, and provided that one interprets the Bayesian formalism inquisitively, humility holds. What is the significance of this conditional statement (assuming that it is true) for humility in general? Let me begin by discussing its relevance for humility on uncountable domains. Above, I created space for a restricted version of humility by appealing to the fact that the argument against humility on uncountable domains did not transfer over to countable ones. But, the argument that I subsequently provided for humility on countable domains does not depend on the assumption that the domain is merely countable, and so, transfers over to the uncountable domain. It follows that the status of humility on uncountable domains is more complicated than it seemed: the too-many-worlds argument tells against it, and the inquisitive argument tells in its favour. This is bad news for everyone involved: the Bayesian who was until now convinced by the too-many-worlds objection finds their stance unsettled, and the inquisitive Bayesian finds that their position commits them to refuting the too-many-worlds objection. So I end this paper on a question mark rather than an affirmative statement.

Let me wrap up by saying a few things about the status of the inquisitive interpretation of the Bayesian formalism. My conclusion in favour of humility on countable domains relies on this interpretation, for which I have provided no argument. Could one then not simply reject it? This turns on the question of what would make an interpretation of Bayesian mathematics wrong or bad. My own view on this is that an interpretation is valuable so long as it describes something of philosophical interest; and it seems clear to me that the inquisitive interpretation does. It seems clear too that the approach I outlined captures something of the Bayesian spirit, if only in that it explains the widespread intuitions in favour of the plausible thought. This answer to the question of what makes an interpret-

[11] For a recent discussion on how this connects to conditionalisation, see Gallow (forthcoming).

ation valuable opens up the possibility that it is interpretive pluralism which will allow us to reconcile our intuition and the case made here for humility restricted to countable domains; and the elaborate views developed by Hájek and Easwaran in response to the too-many-worlds objection on uncountable domains.

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